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Who's Tutoring Who? Reflections from a Field-Based Elementary Mathematics Methods Course

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The purpose of this paper is to describe the perceptions and experiences of preservice teachers enrolled in an elementary mathematics methods course that met each week at an urban Professional Development School. The preservice teachers tutored a group of the lowest 25% in mathematics for 30 minutes during class each week. Twenty-three sets of reflection letters were analyzed. Four themes emerged: (1) Teaching was much harder than simply knowing the content to be taught, (2) Using physical manipulatives during instruction was imperative, (3) Focusing on student error patterns drove instructional interventions, and (4) Understanding how the complexity of students' lives can inhibit learning created empathy. The four themes all reflected preservice teachers' emerging pedagogical content knowledge and suggested the value of the embedded tutoring model.

Keywords: elementary mathematics methods course, pedagogical content knowledge, preservice teachers, Professional Development School (PDS), tutoring

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Introduction

Effective teaching requires knowing more than just content knowledge. Content knowledge is clearly important, but it is only one piece necessary to be an effective classroom teacher. Faculty who teach elementary mathematics methods courses struggle to help preservice teachers (PSTs) understand the differences between *doing* the mathematics and being able to *teach* the mathematics to a diverse group of young learners (Handal, 2003). Particularly in elementary mathematics education, this issue is compounded when PSTs start a methods course feeling uncomfortable with the mathematics content (Bursal & Paznokas, 2006).

Further challenges can occur when PSTs are forced to reconcile competing advice from university faculty and school-based directing teachers while engaging in field experiences. When conflicts arise from contradictions between directing teachers' classroom methods and university course methods, PSTs often conform to the practices of directing teachers (Zeichner & Liston, 1987). Then, during their culminating internship and beginning years of in-service teaching, these PSTs will face a multitude of other challenges: diverse student populations, classroom management routines, lesson planning, parent demands, high stakes testing and accountability, school safety concerns, and cyclical changes in curriculum standards (Boote, 2007; Tynjälä, 2011). In light of these pressing demands, it is easy to understand why PSTs sometimes see little value in their special methods courses.

Recognizing these tensions, Shulman (1986) differentiated among types of knowledge that teachers need to be effective in their practice. While these knowledge types have been categorized in many ways (see Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008; Tamir, 1988), this paper uses the construct of *pedagogical content knowledge* to describe and highlight how an embedded tutoring model within an elementary mathematics methods course effected the experiences of PSTs.

Building on expert-novices studies, Shulman (1986) claimed that what differentiated the expert teachers from their counterparts was their understanding of students' potential struggles with learning challenging concepts. Prior to these cognitive studies of teaching in the 1980s, researchers recognized the importance of teachers' knowledge of the content they taught as well as general pedagogical knowledge necessary for teaching. Shulman's neologism, *pedagogical content knowledge*, acknowledged the special expertise that emerges when content knowledge is combined with pedagogical knowledge. This nuanced understanding of the content enabled teachers to sequence and organize instruction to improve learning. "Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue" (Shulman, 1987, p. 8).

The purpose of this paper is to describe the perceptions and experiences of PSTs enrolled in an elementary mathematics methods course that met each week at an urban Professional Development School (PDS). PSTs' reflection letters, written after each week's one-on-one tutoring sessions, were analyzed and coded for themes. Four themes were found: teaching was much harder than simply knowing the content to be taught, using physical manipulatives during instruction was powerful, focusing on student error patterns drove instructional interventions, and understanding how the complexity of students' lives inhibit learning created empathy.

Pedagogical content knowledge provided a unifying construct to connect the themes that emerged from the data and explained the impact of one-on-one tutoring during a special methods course (Shulman, 1986).

The Embedded Tutoring Model

I taught the elementary mathematics methods courses for three semesters at my University before moving it to the PDS. Prior to moving the course, my PSTs presumed that mere knowledge of elementary mathematics was enough to teach it effectively but, somewhat ironically, many also struggled with their own confidence in mathematics. The result was that some students demonstrated annoyance with my efforts to explain in detail the methods of instruction and how to use various manipulatives to support student learning. The annoyance from a few students created a tone in the class that affected everyone's learning. When meeting the class at our University without the tutoring, I spent most of my energy trying to convince PSTs that teaching mathematics in the elementary schools was actually very challenging. No matter how compelling I tried to be, many did not believe me. They often mistook being able to *do* the elementary mathematics as being able to proficiently *teach* the elementary mathematics. Anyone who has been an elementary school teacher for very long should understand the difference between Pedagogical Content Knowledge and Content Knowledge (Shulman, 1987); but this was not the case with my PSTs. So, instead of staying frustrated at their disbelief of how hard teaching can be without a sound understanding of instruction and curriculum, I elected to provide an authentic way for them to practice the mathematical pedagogy, methods, and materials I was sharing in class with real elementary students.

In Fall 2011, I began serving as Professor in Residence at an urban PDS in partnership with my University. This affiliation allowed me an opportunity to house my 4-credit elementary mathematics methods course on-site at the school. Having access to their elementary students during our scheduled course time was one of the biggest advantages. In Spring 2012, I embedded a tutoring model that allowed each of my PSTs to work one-on-one with the same elementary student for the first 30 minutes of class each week.

The relationship between my University and the PDS was guided by nine *Essentials*, the general qualities of a Professional Development School determined by the National Association of Professional Development Schools (NAPDS, 2008). The tutoring model in my course aligned with two of the *Essentials*:

> 2. A school–university culture committed to the preparation of future educators that embraces their active engagement in the school community; and
> 4. A shared commitment to innovative and reflective practice by all participants. (pp. 2-3)

The tutoring model benefited the school by providing high quality tutoring to some of its struggling learners. At the same time, this model provided PSTs an opportunity to practice innovative instructional methods. Finally, the required weekly reflections following the tutoring sessions encouraged reflective practice.

The school's principal used data from the District's mathematics benchmark tests and Florida Comprehensive Achievement Tests (FCAT) to identify 3rd, 4th and 5th graders

representing the lowest 25th percentile in mathematics achievement. The same data sets were used to identify the domains of mathematics that became the focus of tutoring, whole number place value and base-ten number sense. Students participating in tutoring were pulled out of class each week and met their tutors in the school's cafeteria or, weather permitting, on picnic tables in the courtyard. While field work is common in methods courses and PST education programs, this model was unique because the tutoring component was integrated into the course meeting time instead of being an outside-of-class assignment. Having an opportunity to apply and practice the concepts and skills they were learning in class was an invaluable experience.

The PDS had approximately 450 students enrolled in K-5th grades. There was a high rate of student turnover/transience at this school. More than three quarters of the students lived in low-income housing near the school and was truly a neighborhood school since no busses were used for transportation. It was identified as a Title 1 school on account of having more than 85% of students qualify for free and reduced lunch. In addition, 25% of students were labeled as English Language Learners with an additional 25% having come out of receiving English Speakers of Other Languages services. Impressively, this school had earned an A grade from the Florida Department of Education for five years running before it was given a B grade in 2013.

Immediately following the tutoring sessions each week, PSTs were asked to write reflections about their experiences using the following prompts:

- What did you teach today?
- What instructional methods or manipulatives that we have practiced/used in class did you use?
- What worked well?
- What did not work well?
- Do you have any anecdotes to share?

The reflective writing required about 15 minutes. Another 15 minutes was allotted for whole group discussion of their experiences before transitioning to the day's scheduled coursework. After the final tutoring session, PSTs wrote a final reflection, re-read all previous reflections (about 10 each semester), and wrote a summative reflection to consolidate their thinking about the experience as a whole. Reflections were graded for completion, as evidence of completion of the tutoring activity, and were not graded on the content of the reflections. Not grading the content of the reflection allowed PSTs to reflect honestly and have a more accurate record of their experiences with the student they tutored over the course of the semester.

The primary data for this paper were taken from 23 sets of reflections for one semester to better understand how the embedded tutoring model affected PSTs' perspectives. In addition, field notes and photographs were recorded during tutoring. These were collected primarily for the PSTs to include in their professional portfolios, but they also provided triangulating data that corroborated the reflections. For this paper, analysis of the reflections focused on themes found across a large number of participants: (1) Teaching is much harder than simply knowing the content to be taught, (2) Using physical manipulatives during instruction is imperative, (3) Focusing on student error patterns helps drive instructional interventions, and (4) Understanding how the complexity of students' lives can inhibit learning creates empathy. In the Discussion, I return to explore how pedagogical content knowledge links the themes that emerged from the

data. Specifically, I argue that tutoring struggling learners helped my PSTs to value the pedagogies I taught within my elementary mathematics methods course.

Teaching REALLY is harder than I thought, Professor

Casey (a pseudonym) appreciated the opportunity to practice what she was learning in class with a real elementary student. She had never experienced this type of tutoring model in any of her other education classes. After a semester of working with Shelly, a 3rd grader, Casey reflected:

> I really enjoyed the tutoring this semester. I had never done anything like this. Helping students learn and better understand concepts they are struggling with is an area I personally need work in. [Tutoring Shelly] allowed me to learn how the strategies [we learned in class] are applied in real life. Also, when one explanation didn't help Shelly, it forced me to figure out what other ways I could help her, and I improved and increased my instructional strategies.

Casey pointed out a powerful tenet of this model. When my PSTs returned to our methods class after tutoring, they used the time to seek feedback from each other when their instruction had fallen flat. I particularly liked when they challenged me by sharing when strategies and methods I had preciously taught simply did not work for them. These opportunities helped me grow and better understand ways to improve my instruction, and it also encouraged professional dialogue between my students in the methods classroom.

Natalie, like Casey, was thankful for the opportunity to practice our course objectives with real elementary students. Too often in methods courses, the time gap between when PSTs learn and process content pedagogy in the university classroom and when they can adequately apply it during internship or when they are initially hired in their own classroom is too long. This can be a weakness in any education program. Natalie was one of the most humble of my PSTs. It is not surprising that the 3rd grade student that she tutored made one of the highest learning gains that semester. Natalie reflected:

> I do believe that the tutoring was a valuable experience, at least for myself, if not for my student. I feel that I learned so much more from [our methods] class since I could almost immediately apply what I learned to an actual student. Instead of just filing the activities/strategies away for future use, I was actually able to see which ones were most effective, and which ones I need to work on in order to teach them effectively.

While this excerpt from Natalie's final reflection clearly expressed this theme, all but two PSTs wrote variations describing how the tutoring experience made a profound difference in their understanding of course objectives. The remaining two PSTs' reflections were positive but did not contain sufficient details to be included.

Natalie's humility made it easier for her to use what she was learning in class to make instructional changes during tutoring. When her student was having trouble understanding place value, Natalie knew from class discussions and course readings (e.g., Van de Walle, Karp, & Bay-Williams, 2013) that it was imperative to concretely model what we mean by exchanging 10 ones for 1 ten or vice versa.

The Power of Concrete Manipulatives

Using concrete manipulatives to model mathematical concepts is a central tenet of research-based mathematics instruction (Carbonneau, Marley, & Selig, 2013; Moyer, 2001), but because few PSTs have seen manipulatives used effectively, they often dismiss their value and importance. One reason the PSTs have not seen them modeled is because the in-service teachers in the district felt pressure to keep up with the district's pacing guide. Distributing manipulatives, teaching students how to use them, and dealing with potential classroom management issues all take time. Consequently, teachers are often unwilling to sacrifice the time needed to use them, so these manipulatives do not get used. However, my PSTs who experienced the embedded tutoring model quickly came to appreciate their significance and power.

In addition to the required tutoring experience within the course, each PST also created a *Weapons of Mathematics Instruction* toolkit (see Table 1), an idea shared with me by my late colleague and friend, Dr. Cassandra Etgeton. These toolkits were used during each tutoring session to allow for hands-on instruction and an organized way to address students' learning needs (see Figure 1).

Table 1
Required Items for Weapons of Math Instruction Toolkit

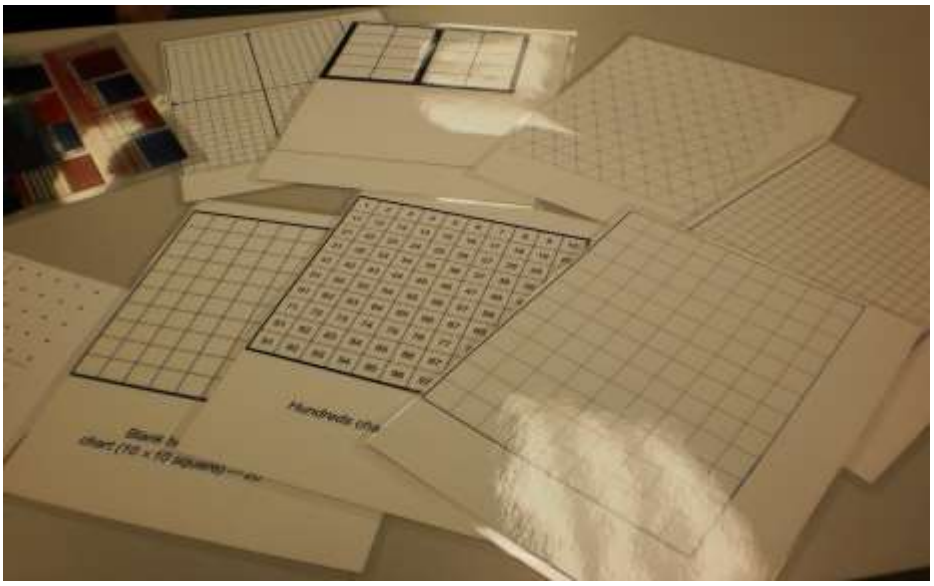
1. Ruler with both inches and centimeters
2. Measuring tape with both inches and centimeters (find in the sewing aisle)
3. Empty egg carton from 1 dozen eggs (no holes in the top)
4. 2 different kinds of dried beans housed in a plastic baggie (e.g., red beans and black-eyed peas)
5. Assortment of empty plastic baggies (2 snack-size, 3 sandwich-size, 4 quart-size, and 2 gallon-size)
6. Assortment of index cards (sizes: 3x5, 4x6, & 5x8; white and assorted colors of each)
7. Assortment of sticky notes (sizes: 3x3 & 4x6; any color)
8. 500 craft sticks bundled in sets of 10 with a small rubber band; put 10 bundles in a plastic quart-sized baggie (total of 5 quart-sized baggies, 50 small rubber bands, 500 craft sticks)
9. 5 sets colored Cuisenaire Rods printed on cardstock. Cut out sets of colored rectangles not individual squares.
10. 1 copy of "Working with Cuisenaire Rods" PDF
11. Plastic Geoboard and colored rubber bands (available online and at teacher stores)
12. At least 4 copies of Geoboard recording sheets on copy paper
13. 1 copy of "Working with Geoboards" PDF
14. 2 sets of tangrams made from the tangram pattern. Keep these in separate bags or make of two different colors.
15. 1 copy of "Working with Tangrams" PDF
16. Base ten blocks printed with 2 different colors. 30 small squares in white, 30 small squares in a different color; 30 rods (rods of ten small squares) in white, 30 rods in a different color; and 10 flats (flats of 100 small squares) in white, 10 flats in a different color. CUT OUT small squares, rods, and flats and paperclip like units together. Suggestion: paperclip sets of 10 small squares together.

17. 3 sets colored and labeled fraction strips *(1/2, 1/3, 1,4, etc.).* Keep 1 set whole—no cutting at all. For the 2nd set, cut all individual fraction pieces apart. Use small paperclips to keep same fraction units together. For the 3rd set, cut horizontal fraction rows apart, keeping like fractional pieces connected.
18. One set of colored fraction circles, unlabeled. CUT APART into fractional parts.
19. One set of square fractions printed on transparencies. Cut individual squares out.
20. 4 sets of colored pattern blocks. Cut all pieces apart.
21. 1 copy of "Working with Pattern Blocks" PDF
22. Dice
23. Place-value mats (1 set laminated, 3 sets paper copies, paper-clip each set)
24. 1 cm square grid
25. Blank Hundreds chart and Filled-in Hundreds chart.
26. Small white board (make sure it fits in your carrying container)
27. Roll of adding machine paper
28. Pencils, pens, crayons, markers, eraser, glue, tape, scissors, and paper clips
29. Notebook paper (at least 25 sheets) and copy paper (at least 25 sheets)
30. Deck of playing cards
31. Calculator
32. 40 small white paper plates & 2 different color circle stickers (200 of one color and 50 of another color)
33. House all of this in a carrying container that will hold everything without having to bend anything.

Note. Many of the materials had corresponding blackline masters that were available via the University's Learning Management System, *Blackboard*.

Figure 1 Weapons of Math Instruction Toolkit Components


JOKER



Ellie noticed right away the struggle between using physical manipulatives with problem solving to teach mathematical concepts compared to relying on direct instruction to teach procedures and algorithms. After Ellie tutored Dana, a 4th grader, she reflected,

> I [see] the power that standard algorithms hold on students. Dana was resistant to using materials at first because it wasn't fast (emphasis in original). However, when she used the materials, she most often got the answer correct... the more we used materials, the more Dana wanted to use them.

This was an important insight, because the notion of a "standard algorithm" also held a lot of power over my PSTs. Many mistakenly thought that the main goal of mathematics instruction

was to teach the ONE correct procedure, but Ellie's experience with Dana highlighted the importance of encouraging students to use concrete materials to model and explore abstract concepts. This excerpt also showed that at least one elementary student had, like many of their teachers, learned to resist concrete manipulatives, because they were too time consuming.

Briane tutored Claire, a 3rd grader, and also reflected on the effectiveness of physical manipulatives, especially base-ten materials (see Figure 2).

Figure 2. Base-Ten Manipulatives Used During Tutoring

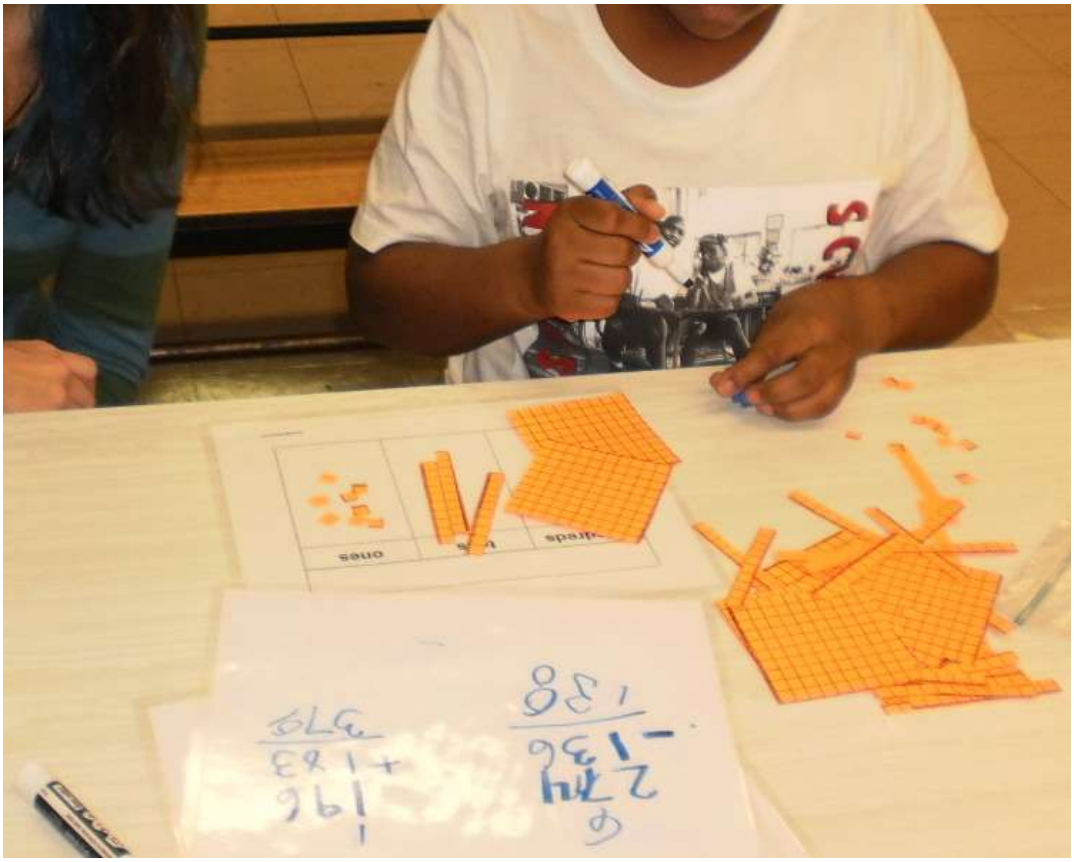


Figure 2. Examples of the base-ten manipulatives used during tutoring. Grouped craft sticks in baggies are referred to as *groupable* and base-ten blocks arranged in single squares (a.k.a. a unit), tens (a.k.a. a rod), and hundreds (a.k.a. a flat) are referred to as *pre-grouped*.

She mentioned a connection to the course textbook (Van de Walle et al., 2013) and felt empowered by knowing the pedagogical content knowledge that informed Claire's developing mathematical understanding. Then, she connected this developmental concept to an appropriate model of instruction.

> Today was a great day for me to realize how important it is to teach conceptually and with manipulatives. I used the activity from Van de Walle where I had Claire add one ten and subtract one ten from a given number. I thought at first that this might be an easy task, but I understood why it might be confusing when she first added the number to the next place value. [376 + 1 = 1,376] When I used my poker chips to help her understand the place value, it clicked for her. That's why I think Claire is in the *position names level*.

During our class discussion after this particular tutoring session, a classmate suggested to Briane that she use a base-ten model with Claire the next time they met. In the classmate's opinion, the base-ten blocks would provide a more concrete representation to show the true ratio occurring in our place value system. Following the next tutoring session, Briane reflected:

> Today I used base 10 blocks and the place value chart that Liz suggested. Every time I asked Claire to use the blocks to add, she wanted to turn to the traditional addition and carrying. I asked her why she carried from ones to tens and she didn't know. So, I explained, "Ten ones equals one ten." Next week, I want to work more on that concept because we didn't have enough time left when we started that. Next week. I want to do similar problems with subtraction.

Briane combined data-driven decision-making and research-based instruction, demonstrating a degree of sophistication that was impressive for a PST.

Formative Assessments to Inform Instruction

One of the course's critical tasks required PSTs to analyze and identify error patterns from a range of student tests using a required text (Ashlock, 2010). Afterwards, they must create a conceptual lesson plan involving manipulatives to remediate the error pattern. In the on-campus sections of the course, PSTs often fail to appreciate the value of this assignment. By contrast, Jackie had the opportunity to work with Kandy, a 5th grader. She reflected:

> This experience was very beneficial. It was great to learn how to teach conceptually with our students. It was also great to make connections using our Ashlock text because you could see actual error patterns occurring and understand firsthand what Ashlock was describing.

Since Jackie elected to use base-ten blocks in her remedial lesson plan of the course's critical task, she had previous experience to use it with Kandy during tutoring. After one particular session, she reflected:

> I am happy because today I feel like I made progress. Kandy did not understand place values last week very well. Today I used the hundreds, tens, and ones chart to break three digit numbers apart (see Figure 3). Then, we practiced making numbers and adding numbers with base 10 blocks. She definitely had a good understanding of place value

after this which was great. I started on multiplication but time ran out. I was glad today that I used my base ten blocks because I feel like they really benefitted Kandy.

Figure 3. Using Base-Ten Manipulatives to Remediate Error Patterns

Figure 3. A place-value chart and pre-grouped base-ten manipulatives provide concrete models and support to help students understand the errors they are making in common algorithms.

Place value and base-ten number sense seemed to be the mathematics strand that the elementary students struggled to understand the most. It also accounted for their gaps in understanding multiplication, division, fractions and percents (see Figure 4).

Figure 4. Using Fraction Bars to Understand Rational Numbers

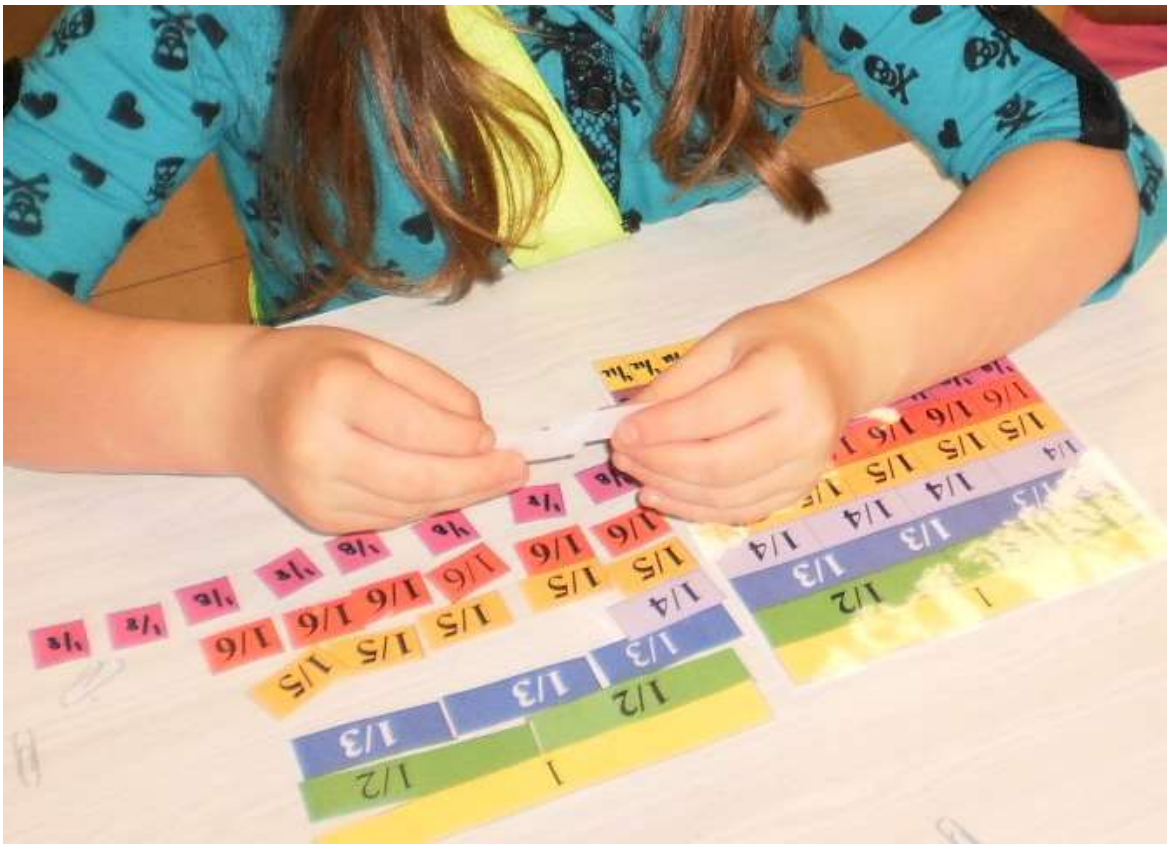


Figure 4. Students used fraction bars cut into fractional sections to understand how to add and subtract fractions with like and unlike denominators as well as to compare fractions.

However, Jackie was able to diagnose Kandy's difficulties with addition as stemming from a deeper confusion about the base-10 number system. Jackie saw in Kandy's work the same kinds of error patterns that she had previously identified in the Ashlock (2010) textbook and error pattern analysis. More generally, the PSTs were able to recognize misunderstood algorithms that surfaced after making deeper conceptual connections. PSTs' ability to make instructional decisions in the moment prevented students from practicing and reinforcing computational errors.

Tutoring as a Snapshot of Real Life

Finally, this tutoring model provided PSTs with a snapshot of some of the challenges that naturally occur in schools. Things happen, life gets in the way, and the lesson plan you've spent so much time preparing falls apart for no apparent reason. Melanie tutored Shay, a 3rd grader, and had more challenges than other PSTs in class. Melanie invited the challenge because she saw it as an opportunity to get a head start working with students of varying ability levels. In her summative reflection, Melanie wrote:

> I laughed a little when I was reading my summaries because I was on a roller coaster ride. Some weeks were great and then the next week was not so great. It was a challenge. I felt defeated some weeks, but I had to keep in mind that to get through to Shay, I needed more than 30 minutes once a week....I found it really depended on what happened at home or in her classroom [before coming to tutoring]. I really feel that the tutoring was an enlightening experience for me. Not every student will be a great success right away. It takes time and many methods to reach all students.

Most teachers in public schools teach students with a range of ability levels. Teacher educators need to prepare PSTs for a variety of learners in their future classrooms.

Most of the students who received tutoring thought the 30 minutes with their tutors was a highlight of their week. Missy experienced this while tutoring Barb, a 3rd grader. For most of their tutoring sessions, Missy helped Barb understand how the position of numbers affected their value. Missy noticed such an improvement in Barb's ability to discuss mathematics concepts that she wrote about it in her reflection.

> Today was a great day working with Barb because she is finally understanding place value conceptually and using it for multiplication. She is using the think-aloud strategy and is using much more sophisticated vocabulary when thinking and solving math problems. I can tell she has been working very hard on trying to understand.

Missy's last reflection on tutoring provided an example of how much it can mean to both a PST and an elementary child to have an opportunity to spend time working together over the course of a semester.

> Barb has definitely impacted my life in such a positive way. She is such a sweet and caring child. Today was my birthday and she remembered it. When I walked into the cafeteria, she had everyone singing for me. She did not like that it was our last [tutoring] day, and I'm pretty sure she gave me at least 12 hugs! She also told me before I left that she was very thankful for me helping her. I am as sad as her just knowing that today was our last day together.

Discussion

The themes that emerged from my PSTs' reflections suggested that they learned at least as much from the students they were assigned to tutor as the elementary students learned from them. PSTs often came to my elementary mathematics methods course believing that because they can *do* the mathematics themselves, it will be easy to *teach* the mathematics. In the on-campus sections of the course, it is often difficult to disabuse them of these beliefs (Handal, 2003). However, when confronted with real children who are struggling to understand and do the mathematics, the content of my course suddenly becomes salient. As a result, they value physical manipulatives to model mathematical ideas, they actively analyze student errors for insights about appropriate instructional interventions, and better appreciate the complexity of students' lives that sometimes inhibit learning. In short, these tutoring experiences helped my PSTs to appreciate the value of pedagogical content knowledge and, in turn, helped them to learn the necessary pedagogical content knowledge (Shulman, 1986). In their final summative reflections, almost everyone echoed the belief that manipulatives encouraged students to engage with mathematics and could easily occur in a problem-centered lessons (see Figure 5) (see also Keller, 2011).

Figure 5. Using Manipulatives During Problem Solving

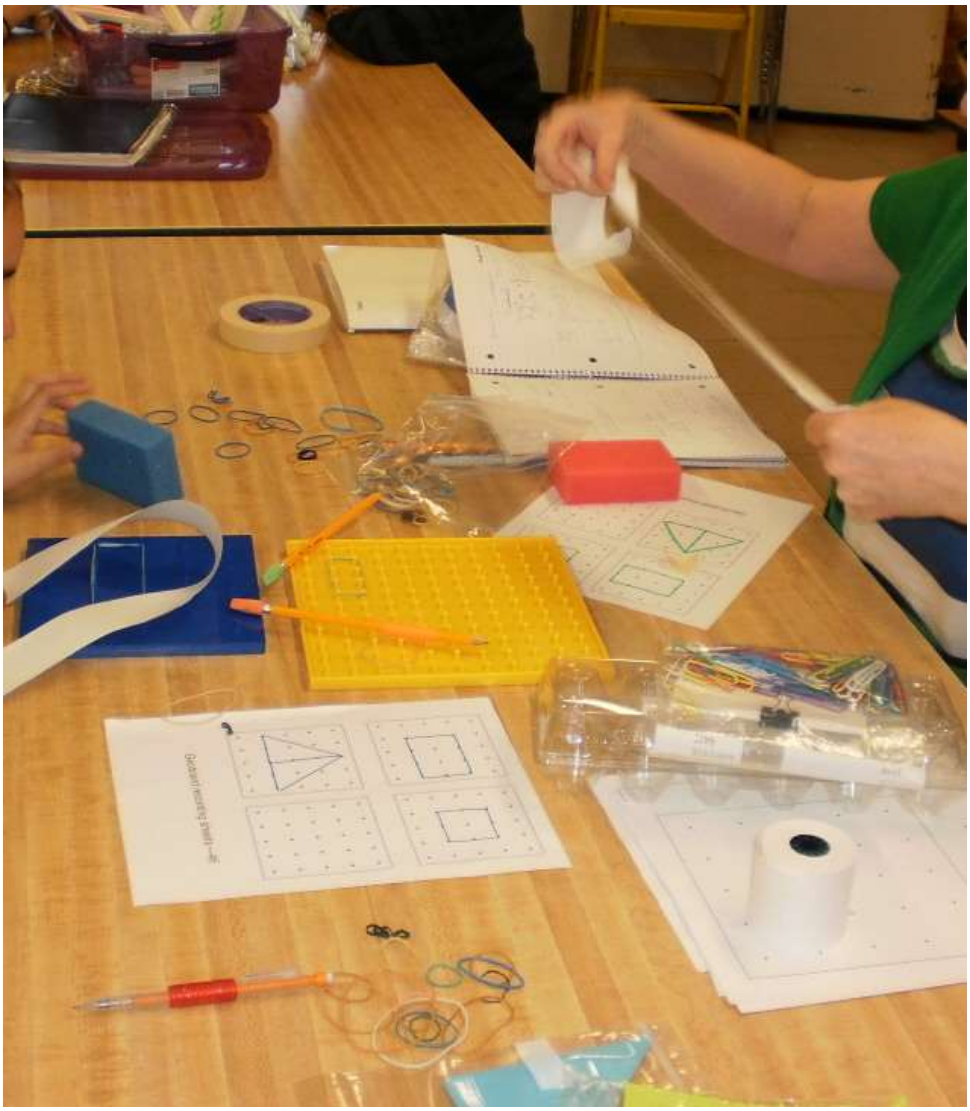


Figure 5. Mathematics manipulatives encourage problem solving behaviors and provide students with multiple entry points into the problem.

For all of the advantages of this embedded tutoring model, there were also some challenges. Organizing the embedded tutoring model required a considerable amount of coordination and planning at the school, and the school administrators were already very busy. In addition, the classroom teachers had to be willing to relinquish their students for at least 30 minutes of classroom instructional time. For this to happen smoothly, these classroom teachers had to believe that the tutoring was at least as effective as what the students were missing in class.

For the first year at the school, I did not have an assigned room and was often at the mercy of available space. It was challenging for me to transport the many materials required for this kind of course. It was also challenging for the PSTs to travel to an off-campus site. Finally, adding this component to my class forced me to compress the remaining instructional time. Overall, however, the improved elementary students' test scores, improved participation in their classes, and increased motivation to learn mathematics far outweighed the challenges and confirmed the benefits of the tutoring and PDS model (NAPDS, 2008).

Incorporating a one-on-one tutoring opportunity for PSTs enrolled in an elementary mathematics methods course was an effective way to focus their attention on the important but often abstract components of pedagogical content knowledge. Teaching my methods course at a local elementary school allowed me to embed tutoring into the course itself instead of making it an extra out-of-class field assignment. Working with elementary students throughout the semester provided an authentic opportunity to practice the methods and activities taught in the course. This strategy helped bring what is often taught as best practice and theory inside university methods courses to a tangible and concrete level for PSTs. In this age of differentiated instruction and accountability, this tutoring model offered an effective approach for using hands-on instruction to support learning for both university and elementary students.

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